

CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, July/August 2021 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions.

- 1 a. Obtain the Fourier series for the function,

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ (08 Marks)

- b. Find the constant term and first two harmonics in the Fourier series for $f(x)$ given by the following table:

| | | | | | | | |
|------|-----|---------|----------|-------|----------|----------|--------|
| x | 0 | $\pi/3$ | $2\pi/3$ | π | $4\pi/3$ | $5\pi/3$ | 2π |
| f(x) | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

(08 Marks)

- 2 a. Expand $f(x) = \sqrt{1 - \cos x}$ in $0 \leq x \leq 2\pi$ in a Fourier series. Evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots$

(08 Marks)

- b. Obtain the Fourier series for $f(x) = |x|$ in $(-\ell, \ell)$ and hence evaluate $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

(08 Marks)

- 3 a. Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and hence deduce that

$$\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$$

(06 Marks)

- b. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$ where $m > 0$.

(05 Marks)

- c. Find the z-transform of (i) $(2n-1)^2$ (ii) $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$

(05 Marks)

- 4 a. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$. Hence deduce $\int_0^{\infty} \frac{\sin ax}{x} dx$.

(06 Marks)

- b. Find the inverse z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$.

(05 Marks)

- c. Solve the differential equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ using z-transform method.

(05 Marks)

- 5 a. Find the coefficient of correlation and the two lines of regression for the following data:

| | | | | | | | | | | |
|---|---|---|----|---|----|----|----|----|----|----|
| x | 1 | 3 | 4 | 2 | 5 | 8 | 9 | 10 | 13 | 15 |
| y | 8 | 6 | 10 | 8 | 12 | 16 | 16 | 10 | 32 | 32 |

(06 Marks)

- b. Fit a curve of the form $y = ae^{bx}$ to the following data:

| | | | | | |
|---|-----|-----|-----|------|------|
| x | 77 | 100 | 185 | 239 | 285 |
| y | 2.4 | 3.4 | 7.0 | 11.1 | 19.6 |

(05 Marks)

- c. Use Regula Falsi method, find the root of the equation $x^2 - \log_e x - 12 = 0$.

(05 Marks)

- 6 a. The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.5x$. Find:

- (i) Means of x
 (ii) Means of y
 (iii) The correlation coefficient

(06 Marks)

- b. Fit a parabola $y = a + bx + cx^2$ to the following data:

| | | | | | | | |
|---|------|------|------|------|------|------|------|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 4.63 | 2.11 | 0.67 | 0.09 | 0.63 | 2.15 | 4.58 |

(05 Marks)

- c. Use Newton-Raphson method to find the real root of $3x = \cos x + 1$, take $x_0 = 0.6$ perform 2 iterations.

(05 Marks)

- 7 a. Find the cubic polynomial by using Newton forward interpolating formula which takes the following values.

| | | | | |
|---|---|---|---|----|
| x | 0 | 1 | 2 | 3 |
| y | 1 | 2 | 1 | 10 |

(06 Marks)

- b. Apply Lagrange's formula inversely to obtain a root of the equation $f(x) = 0$ given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, $f(42) = 18$.

(05 Marks)

- c. Use Weddle's rule to evaluate $\int_0^{\pi/2} \sqrt{\cos \theta} d\theta$ dividing the interval $\left[0, \frac{\pi}{2}\right]$ into six equal parts.

(05 Marks)

- 8 a. A survey conducted in a slum locality reveals the following interpolating information as classified below:

| | | | | | |
|--------------------------|----------|-------|-------|-------|-------|
| Income/day in rupees : x | Under 10 | 10-20 | 20-30 | 30-40 | 40-50 |
| Number of persons : y | 20 | 45 | 115 | 210 | 115 |

Estimate the probable number of persons in the income group 20 to 25.

(06 Marks)

- b. Using Newton divided difference formula fit an interpolating polynomial for the following data:

| | | | | |
|------|---|----|----|-----|
| x | 0 | 1 | 4 | 5 |
| f(x) | 8 | 11 | 68 | 123 |

(05 Marks)

- c. Using Simpson's $1/3^{\text{rd}}$ rule evaluate $\int_0^1 \frac{x^2}{1+x^3} dx$ taking four equal strips.

(05 Marks)

- 9 a. Find the extremal of the functional $I = \int_0^{\pi/2} (y'^2 - y'^2 - 2y \sin x) dx$ under the conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$. (06 Marks)
- b. If $\vec{F} = x^2\mathbf{i} + xy\mathbf{j}$ evaluate $\int_c \vec{F} \cdot d\vec{r}$ from (0, 0) to (1, 1) along
 (i) the line $y = x$ (ii) the parabola $y = \sqrt{x}$ (05 Marks)
- c. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis gives a minimum surface area. (05 Marks)
- 10 a. Verify Green's theorem in a plane for $\oint_c (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. (06 Marks)
- b. Using divergence theorem evaluate $\int \vec{A} \cdot \vec{n} ds$ where $\vec{A} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and s is the surface of the surface $x^2 + y^2 + z^2 = a^2$. (05 Marks)
- c. Find the geodesics on a surface given that the arc length on the surface is $s = \int_{x_1}^{x_2} \sqrt{x(1+y'^2)} dx$. (05 Marks)

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15ME32

Third Semester B.E. Degree Examination, July/August 2021 Material Science

Time: 3 hrs.

Max. Marks: 80

Note : 1. Answer any FIVE full questions.
2. Draw neat sketches, wherever required.

- 1 a. Explain the FCC structure. From the basics find its APF. (06 Marks)
b. Differentiate between Edge and Screw dislocation. (04 Marks)
c. Sketch the Stress – Strain diagram for mild steel and mark the salient points. Explain the following properties i) Yield strength ii) Toughness. (06 Marks)
- 2 a. Differentiate between Ductile and Brittle fracture. (05 Marks)
b. With $\epsilon - t$ diagram, explain the various stages of creep. (06 Marks)
c. Explain how SN diagrams are Plotted for a material. (05 Marks)
- 3 a. Explain the Hume – Rothery rules governing for the formation of Substitutional Solid Solutions. (05 Marks)
b. Construct a phase diagram, to the scale on graph sheet for the following details :
Melting point of silver (Ag) = 961°C ; Melting point of Copper (Cu) = 1083°C .
Eutectic temperature = 780°C ; Eutectic composition = 28% Cu.
Maximum solubility of Cu in Ag = 9% at 780°C .
Maximum solubility of Ag in Cu = 8% at 780°C .
Maximum solubility of Cu in Ag = 2% at 400°C .
Maximum solubility of Ag in Cu = 0% at 400°C .
Label all the fields. Determine the following :
i) Solidification process for 20% Cu.
ii) Amount of Eutectic in 20% Cu – 80% Ag alloy at 700°C .
iii) Percentage and composition of the liquid and solid phases in a 20% Ag and 80% Cu alloy at 900°C . (11 Marks)
- 4 a. Sketch the Iron – Carbon equilibrium diagram labeling the different fields and write down the equations for Eutectic, Eutectoid and Peritectic reactions. (10 Marks)
b. What are Solid Solutions? List the various types of Solid solutions with sketch. (06 Marks)
- 5 a. Explain how TTT curves are plotted. (06 Marks)
b. Enumerate the complete classification of Heat treatment process. (05 Marks)
c. Differentiate between Annealing and Normalizing. (05 Marks)
- 6 a. Explain the Flame hardening process, with neat sketch. (06 Marks)
b. Explain the Composition, Properties and Microstructure of
i) Grey Cast Iron ii) SG Iron. (06 Marks)
c. Explain the Age hardening of Aluminum – Copper Alloys. (04 Marks)
- 7 a. Explain the following Mechanical properties of Ceramics :
i) Tensile strength ii) Hardness iii) Compressive strength iv) Modulus of elasticity. (04 Marks)

15ME32

- b. Enumerate the classification of Ceramic fabrication techniques. (05 Marks)
- c. Explain with a neat sketch, the Compression Molding of Polymers. (07 Marks)
- 8 Explain the following :
- a. Shape Memory alloys. (06 Marks)
- b. Fiber Optic materials. (05 Marks)
- c. Biological application of Smart Materials. (05 Marks)
- 9 a. What are Composites? Enumerate the complete classification of Composites, with sketches. (08 Marks)
- b. Explain with neat sketch, the Hand lay up process. (08 Marks)
- 10 a. With a neat sketch, explain the Pultrusion process. (06 Marks)
- b. Write a note on Hybrid Composites. (05 Marks)
- c. Calculate modulus of elasticity and the tensile strength of the following Composite material stressed under isostrain conditions. The composite consists of a continuous glass – fiber – reinforced – epoxy resin produced by using 60% by volume of E – glass fibers having a modulus of elasticity of $E_f = 72$ GPa and a tensile strength of 2400 MPa and a hardened epoxy resin with a modulus of elasticity of $E_m = 3$ GPa and tensile strength of 62MPa. (05 Marks)

CBCS SCHEME

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1KT16ME02A

15ME33

Third Semester B.E. Degree Examination, July/August 2021 Basic Thermodynamics

Time: 3 hrs.

Max. Marks: 80

Note: 1. Answer any FIVE full questions.

2. Use of thermodynamic data hand book permitted.

- 1 a. Define thermodynamics. Explain the macroscopic and microscopic approaches. (03 Marks)
b. Define (i) Open system (ii) Closed system (iii) Property. (05 Marks)
c. The temperature T on a scale is given by $T = a \ln x + b$, where a and b are constants and x is thermometric property. The values of x at ice and steam points are 1.83 and 6.78. The corresponding temperatures are assigned values 0 and 100 respectively. Determine temperature when $x = 2.42$. (08 Marks)
- 2 a. Define work according to thermodynamics. (02 Marks)
b. Derive expressions for the displacement work for polytropic process. (06 Marks)
c. Air is compressed from atmospheric pressure and 0.2 m^3 to 5 bar in a polytropic process with an index of compression of 1.4. Calculate the work needed for compression. Had the compression been carried out hyperbolically between the same initial state and the same final pressure, what would be the work needed? (08 Marks)
- 3 a. State the first law of thermodynamic for closed system. (04 Marks)
b. Prove that energy is a property of system. (04 Marks)
c. At the inlet of a nozzle, the enthalpy of working fluid is 3 MJ/kg and the velocity is 60 m/s, At the exit, the enthalpy is 2.76 MJ/kg. The nozzle is horizontal and is insulated. Calculate the velocity at the exit. If the diameter of inlet is 357 mm and density of working fluid at inlet is 5.35 kg/m^3 , determine the mass flow rate. If the density of working fluid is 2 kg/m^3 at exit, determine the exit diameter. (08 Marks)
- 4 a. A Carnot engine has a rated output of 5 kW. The heat supplied is 6 kW. Calculate the efficiency and heat rejected. Also determine the source and sink temperatures, if the difference is 300°C . (06 Marks)
b. With the help of a schematic diagram, show that Kelvin-Planck and Clausius statements of second law of thermodynamics are equivalent. (10 Marks)
- 5 a. Define reversible and irreversible processes. What is the need to define a reversible process as it is not at all practical? (03 Marks)
b. Explain the factors that make a process irreversible. (05 Marks)
c. Prove the basic equation of absolute thermodynamic temperature scale $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ for a reversible heat engine. (08 Marks)

- 6 a. Prove that entropy is indeed a property. (03 Marks)
 b. 1.2 kg of nitrogen at 120 kPa, 300 K is compressed polytropically until the volume reduces by 50%. The index of compression is 1.3 and $C_p = 1.04$ kJ/kgK. Determine the entropy change during compression. (08 Marks)
 c. Draw Carnot cycle on a T-S plot and show that the network is given by $(T_1 - T_2) \Delta S$. (05 Marks)
- 7 a. Air expands in a turbine from 500 kPa, 520°C to 100 kPa, 300°C. The heat loss to the surroundings is 10 kJ/kg, and surrounding is at 20°C. Determine available energy and irreversibility per kg of air. (08 Marks)
 b. Derive Maxwell's relations. (08 Marks)
- 8 a. Draw representative P-T diagram for water and explain the regions. (04 Marks)
 b. Steam initially at 150 bar, 500°C expands reversibly and adiabatically in a turbine to a pressure of 0.1 bar. If the steam flow rate is 600 tons per hour. Determine the work output of turbine. Use steam tables for the properties of steam. (08 Marks)
 c. Steam flows in a pipe at 15 bar. After expanding to 1 bar in a throttling calorimeter, the temperature is found to be 110°C. Determine the quality of steam in the pipe using Mollier chart. (04 Marks)
- 9 a. A mixture of ideal gases consists of 79 kg Nitrogen and 21 kg oxygen at 1 bar. Calculate the (i) Partial pressures, (ii) Equivalent molecular weight of the mixture, and (iii) Equivalent gas constant of the mixture. (09 Marks)
 b. Define (i) Specific humidity and (ii) Relative humidity. (02 Marks)
 c. Atmospheric air at mean sea level and 30°C has a relative humidity of 80% using psychrometric chart, find (i) wet bulb temperature (ii) specific humidity and (iii) dew point temperature. (05 Marks)
- 10 a. What are the limitations of ideal gas equation? (02 Marks)
 b. Estimate the pressure of 100 kg of Nitrogen, which occupies a volume of 0.375 m³ at 175 K using (i) Ideal gas equation, and (ii) Vander Waal's equation. Take Vander Waal's constants $a = 136.6$ kN m⁴/kmol² and $b = 0.0386$ m³/Kmol. (08 Marks)
 c. Determine the density of steam in a boiler at 406°C and 332 bar using generalized compressibility chart. (06 Marks)

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15ME/MA34

Third Semester B.E. Degree Examination, July/August 2021 Mechanics of Materials

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions.

- 1 a. Explain in brief, the terms 'stress' and its types, and 'strain' and its types. (06 Marks)
- b. For a laboratory tested specimen, the following data were obtained:
Diameter of the specimen = 25 mm
Length of the specimen = 300 mm
Extension of specimen under a load of 15 kN = 0.045 mm
Load at yield point = 127.65 kN
Maximum load = 208.6 kN
Length of specimen after failure = 375 mm
Neck diameter at failure = 17.75 mm
Determine: (i) Young's modulus (ii) Yield point stress (iii) Ultimate stress
(iv) Percentage elongation (v) Percentage reduction in area (10 Marks)

- 2 a. A uniformly tapering circular bar having smaller diameter d_1 and larger diameter d_2 at the two ends is subjected to a pull of P . If the length of the bar is 'L', find the total deformation in the bar. (08 Marks)
- b. A stepped bar with varying cross sections is subjected to forces as shown in Fig.Q2(b) below:

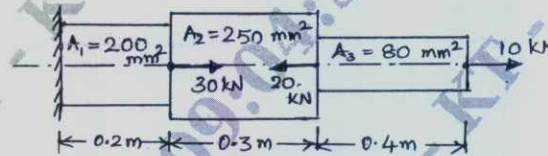


Fig.Q2(b)

- Determine the net deformation in the bar if the Young's modulus of bar material is $2 \times 10^5 \text{ N/mm}^2$. (08 Marks)
- 3 a. When a certain thin walled tube is subjected to internal pressure and torque, the stresses in the tube wall are (i) 120 MPa (tensile - x direction), (ii) 60 MPa (tensile - y direction), (iii) complimentary shear stress of 90 MPa in the directions of (i) and (ii).
Find the normal and tangential stress on the two planes which are equally inclined to stresses in (i) and (ii). (08 Marks)
- b. At a point in a bracket, the stresses on two mutually perpendicular planes are 35 MPa (tensile) and 15 MPa (tensile). The shear stress across these planes is 9 MPa. Find the magnitude and direction of the resultant stress on a plane making an angle of 40° with the plane of first stress. Find also, the normal and tangential stress on the plane. Use Mohr's circle method. (08 Marks)
- 4 a. A thin cylinder of diameter 'd', wall thickness 't' and length 'L' is subjected to an internal fluid pressure of 'p'. Find the equations for circumferential and longitudinal stresses induced in the cylinder material. Draw neat sketches wherever necessary. (08 Marks)
- b. A pipe of 400 mm internal diameter and 100 mm wall thickness carries a fluid at a pressure of 80 MPa. Calculate the maximum and minimum hoop stresses across the section. Also, sketch the distribution of radial and hoop stresses across the thickness of the cylinder. (08 Marks)

- 5 a. What is a beam? With neat sketches, explain briefly the types of beams and the loads they carry. (08 Marks)
- b. A cantilever beam 2m long carries a UDL of 1.5 kN/m over the entire span. It also carries a point load of 2 kN at a distance of 0.5 m from the free (right) end. Draw the SFD and BMD of the beam. (08 Marks)
- 6 a. Derive the differential equation of deflection (Euler-Bernoulli). List the assumptions made in the derivation. (08 Marks)
- b. A uniform I - section beam is 5m long and carries a UDL of 83 kN/m on its entire span. The I - section is 100 mm wide and 150 mm deep. The thickness of flanges is 25 mm each and the web thickness is 12 mm. If the beam is simply supported, determine the bending stress in the beam. (08 Marks)
- 7 a. Derive torsion equation using suitable notations. Draw neat sketches wherever necessary. (10 Marks)
- b. Determine the diameter of a steel shaft which will transmit 90 KW of power at 160 rpm. The maximum shear stress induced is 60 MPa. Find also the length of the shaft if the twist in the shaft must not exceed 1° over the entire length. Take $G = 80 \text{ GPa}$. (06 Marks)
- 8 a. A column of length l , having its moment of inertia as I and Young's modulus E carries a compressive load of P . If the column is hinged at both the ends, find the Euler's buckling load equation for the column. (08 Marks)
- b. A 1.5 m long, circular C/S column of 50 mm diameter has one of its ends fixed in direction and position while the other end is free. Taking a factor of safety of 3, calculate the safe load the column can carry using:
- (i) Rankine formula, with yield stress as 560 N/mm^2 and constant $\alpha = \frac{1}{1600}$.
- (ii) Euler's formula, taking $E = 1.2 \times 10^5 \text{ N/mm}^2$. (08 Marks)
- 9 a. A stepped bar of 1 m length is subjected to an axial pull such that the maximum tensile stress is equal to 150 MPa. Calculate the strain energy stored in the bar if $E = 200 \text{ GPa}$. [Refer Fig.Q9(a)]

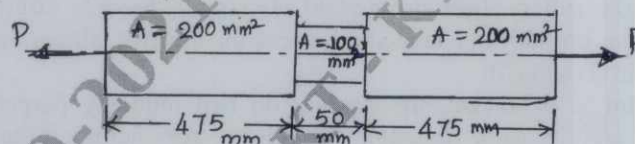


Fig.Q9(a)

- (08 Marks)
- b. Find an expression for the strain energy due to bending of a beam of length 'L', simply supported at the ends and carrying a UDL of $W/\text{unit length}$ over its entire length. The beam is of constant cross-section throughout its length having flexural rigidity as EI . (08 Marks)
- 10 a. In a metallic body, the principal stresses are $+35 \text{ MPa}$ and -95 MPa , the third principal stress being zero. The elastic limit stress in simple tension as well as in simple compression is equal and is 220 MPa . Find the factor of safety based on the elastic limit if the criterion of failure for the material is the maximum principal stress theory. (08 Marks)
- b. A mild steel shaft 120 mm diameter is subjected to a maximum torque of 20 kN-m and a maximum bending moment of 12 kN-m at a particular section. Find the factor of safety according to maximum shear stress theory if the elastic limit in simple tension is 220 MN/m^2 . (08 Marks)

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15MATDIP31

Third Semester B.E. Degree Examination, July/August 2021

Additional Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions.

1. a. Express $\frac{(3+i)(1-3i)}{(2+i)}$ in the form $x + iy$. (06 Marks)
 b. Find the modulus and amplitude of the complex number $1 + \cos \alpha + i \sin \alpha$. (05 Marks)
 c. If $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$, then find $\vec{a} \times (\vec{b} \times \vec{c})$. (05 Marks)

2. a. Prove that $\left[\frac{1 + \cos \theta + i \sin \theta}{1 + \cos \theta - i \sin \theta} \right]^n = \cos n\theta + i \sin n\theta$. (06 Marks)
 b. Find the cube root of $1 + i\sqrt{3}$. (05 Marks)
 c. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar. (05 Marks)

3. a. Find the n^{th} derivative of $e^{ax} \sin(bx + c)$. (06 Marks)
 b. With usual notations prove that $\tan \phi = r \cdot \frac{d\theta}{dr}$. (05 Marks)
 c. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. (05 Marks)

4. a. Find the n^{th} derivative of $\frac{x}{(x-2)(x-3)}$. (06 Marks)
 b. Find the angle between the curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$. (05 Marks)
 c. Given $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (05 Marks)

5. a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$. (06 Marks)
 b. Evaluate $\int_0^{\pi/16} \cos^5(8x) \sin^6(16x) \, dx$. (05 Marks)
 c. Evaluate $\int_1^2 \int_1^3 x y^2 \, dx \, dy$. (05 Marks)

6. a. Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} \, dx$. (06 Marks)
 b. Evaluate $\int_0^{\pi} \frac{\sin^4 \theta}{(1 + \cos \theta)^2} \, d\theta$. (05 Marks)
 c. Evaluate $\int_{-3}^3 \int_0^1 \int_1^2 (x + y + z) \, dx \, dy \, dz$. (05 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 7 a. Find velocity and acceleration of a particle moving along the curve $\vec{r} = e^{-2t} \hat{i} + 2 \cos 5t \hat{j} + 5 \sin t \hat{k}$ at anytime t . Find their magnitudes at $t = 0$. (06 Marks)
- b. If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\nabla\phi$ at $(1, -1, 2)$. (05 Marks)
- c. Show that $\vec{F} = (x + 3y) \hat{i} + (y - 3z) \hat{j} + (x - 2z) \hat{k}$ is Solenoidal. (05 Marks)
- 8 a. Find the unit tangent vector of the space curve $\vec{r} = \cos t \hat{i} + \sin t \hat{j} + t \hat{k}$. (06 Marks)
- b. If $\vec{F} = x^2y \hat{i} + yz^2 \hat{j} + zx^2 \hat{k}$, then find $\text{div}(\text{curl } \vec{F})$. (05 Marks)
- c. Find the constants a, b and c such that the vector $\vec{F} = (x + y + az) \hat{i} + (x + cy + 2z) \hat{j} + (bx + 2y - z) \hat{k}$ is irrotational. (05 Marks)
- 9 a. Solve $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$. (06 Marks)
- b. Solve $\frac{dy}{dx} + y \cot x = \sin x$. (05 Marks)
- c. Solve $\frac{dy}{dx} = \frac{x^2 - 2xy}{x^2 - \sin y}$. (05 Marks)
- 10 a. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. (06 Marks)
- b. Solve $(1 + xy)y dx + (1 - xy)x dy = 0$. (05 Marks)
- c. Solve $x \frac{dy}{dx} + y = x^3 y^6$. (05 Marks)
